CLASSIFICATION [redacted] FOR OFFICIAL USE ONLY

FDD FILE COPY

CENTRAL INTELLIGENCE AGENCY
INFORMATION REPORT

REPORT

STAT

COUNTRY USSR

DATE DISTR. 20 Oct 1948

SUBJECT Physics

NO. OF PAGES 17

UNCLASSIFIED
FEB 2 1955 FDD
FOR OFFICIAL USE ONLY

PLACE ACQUIRED USSR

NO. OF ENCLS. (LISTED BELOW)

DATE OF INFORMATION 1946

SUPPLEMENT TO REPORT NO.

STAT

THIS IS UNEVALUATED INFORMATION FOR THE RESEARCH USE OF TRAINED INTELLIGENCE ANALYSTS

SOURCE Russian periodical, Zhurnal Eksperimental'noy i Teoreticheskoy Fiziki, No 7, 1946. (FDB Per Abs 40T93 -- Translation specifically requested.)

ACOUSTICAL RADIATION OF OSCILLATING BODIES IN A COMPRESSIBLE FLUID

M. D. Khaskind

/NOTE: Numbers in brackets refer to the bibliography/

INTRODUCTION

In this work is carried out a theoretical investigation of the problem of the acoustical radiation of oscillating bodies in a compressible fluid for the most general case of harmonic oscillation of a solid and deformable body.

During the oscillation of the solid body, the hydrodynamic forces acting upon it may be divided into inertial and damping forces.

The inertial forces are expressed linearly through acceleration. In determining the law of motion of the oscillating body, the coefficients of the inertial forces acting on the body play the part of additional masses, of moments of inertia, etc. Hence the coefficients of the inertial forces may be called connected masses, this being a generalization of the existing concept of a connected mass for an infinite and incompressible fluid.

The properties of symmetry of connected masses, occurring in incompressible fluids, remain valid also in the examined case. The numerical values of the generalized connected masses are related to the frequency of oscillation.

The damping forces accounting for continuous expenditure of energy on the formation of acoustical waves are linearly related to the velocities. The same properties of symmetry hold true for the coefficients of damping as for the generalization of the connected masses. On the basis of the formulas obtained for the generalized connected masses and coefficients of damping, accurate and approximate calculations may be made for several concrete cases.

CLASSIFICATION [redacted]

STATE	X	NAVY	X	NSRB		DISTRIBUTION							
ARMY	X	AIR	X										

[redacted] FOR OFFICIAL USE ONLY

~~RESTRICTED~~

[redacted]

I. BASIC EQUATIONS

A body oscillating in a fluid produces around itself a periodical compression and vacuum in the fluid and, hence, leads to the formation of accoustical waves.

The general case of the problem of acoustical radiation of a body of arbitrary form producing small oscillations in an ideal compressible fluid can be reduced to the determination of the potential of velocities $\Phi(x,y,z,t)$ fulfilling the conditions. On the surface of the body S the streamline condition

$$\frac{\partial \Phi}{\partial n} = v_n(M,t), \tag{1.1}$$

obtains, where M is the applied point of the surface S, and v_n the normal component of velocity of any point on the surface S. We shall also consider the normal to the surface S to be directed into the liquid. The function v_n is related to the forward and angular velocities and to the velocity of deformation of the body. We have:

$$v_n(M,t) = V\cdot n + (\Omega \times r_0)n + V_g n = nV + (r_0 \times n)\Omega + nV_g, \tag{1.2}$$

where $V = U_1 e_x + U_2 e_y + U_3 e_z$, is the vector of the velocity of the origin of the coordinates; $\Omega = U_4 e_x + U_5 e_y + U_6 e_z$ is the vector of the instantaneous angular velocity; V_g is the velocity of deformation of the oscillating body at the point M; e_x, e_y, e_z and n are single vectors of the axes of the coordinates and normals to the surface S, and r_0 = OM is the radius vector of point M.

In the total mass of fluid the potential of velocities Φ fulfills the wave equation:

$$\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} + \frac{\partial^2 \Phi}{\partial z^2} = \frac{1}{c^2}\frac{\partial^2 \Phi}{\partial t^2}, \tag{1.3}$$

where S is the velocity of sound determined by the pressure p and the density ρ from the formula:

$$c = \sqrt{\left(\frac{dp}{d\rho}\right)_0} \tag{1.4}$$

In addition to these conditions, the potential of velocities Φ satisfies at infinity the principle of radiation in that the radiated accoustical waves diverge in all directions from the oscillating body, i. e., at large distances from the body, radiated waves develop into diverging spherical waves.

For all further cases let us examine a case of simple harmonic oscillations of a body resulting from a frequency k

$$V = ve^{ikt}, \Omega = \omega e^{ikt}, V_g = v_g e^{ikt}, U_m = u_m e^{ikt} (m = 1, 2, \ldots, 6) \tag{1.5}$$

[redacted]

RESTRICTED

~~RESTRICTED~~

[redacted]

STAT

Here, and later in complex expressions containing the multiple $e^{ikt} (i = \sqrt{-1})$ it is necessary to examine only the real part.

Considering the disturbed oscillating motion of the liquid as constant, let us assume that

$$\Phi(x, y, z, t) = \varphi(x, y, z) e^{ikt}. \tag{1.6}$$

To determine the function $\varphi(x,y,z)$ we have the conditions

$$\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} + \frac{\partial^2 \varphi}{\partial z^2} + \nu^2 \varphi = 0 \text{ beyond } S \ (\nu = k/c) \tag{1.7}$$

$$\frac{\partial \varphi}{\partial n} = n \cdot v + (r_0 \times n) \cdot \omega + n \cdot v_g \text{ on } S'; \tag{1.8}$$

$$\lim_{R \to \infty} R \left(\frac{\partial \varphi}{\partial R} + i \nu \varphi \right) = 0 \ (R^2 = x^2 + y^2 + z^2). \tag{1.9}$$

The last of the above conditions represents the mathematical formulation of the principle of radiation.

For the function $\varphi(x,y,z)$ we have an equation giving a generalization of green's formula [1]:

$$\varphi(x, y, z) = -\frac{1}{4\pi} \iint_S \left(\frac{e^{-i\nu r}}{r} \frac{\partial \varphi}{\partial n} - \varphi \frac{\partial}{\partial n} \frac{e^{-i\nu r}}{r} \right) dS, \tag{1.10}$$

where

$$r = \sqrt{(x - \xi)^2 + (y - \eta)^2 + (z - \zeta)^2}, \tag{1.11}$$

where the points P (ξ, η, ζ) move along the surface S and the external direction of the normal is followed.

On the basis of formula 1.10 it is easy to determine the asymptotic nature of the disturbed motion of the fluid with larger values of R. For this purpose, let us introduce the spherical coordinates of R, θ, ψ, with the center at the origin of the coordinates, i. e.,

$$x = R \sin\theta \cos\psi, \quad y = R \sin\theta \sin\psi, \quad z = R \cos\theta \tag{1.12}$$

[redacted]

[redacted]

~~RESTRICTED~~

STAT

~~RESTRICTED~~

Then, in the case of values of R and r we have:

$$r = R - \chi + O\left(\frac{1}{R}\right), \tag{1.13}$$

where

$$\chi = \sin(\xi\cos\psi + \eta\sin\psi) + \zeta\cos\theta \tag{1.14}$$

and $O(a)$ denotes terms of the order of smalless a.

Thus from formula 1.10 we obtain the asymptotic formula

$$\varphi = Q(\nu, \theta, \psi)\frac{e^{-i\nu R}}{2\pi R} + O\left(\frac{1}{R^2}\right), \tag{1.15}$$

where

$$Q(\nu, \theta, \psi) = \iint_S e^{i\nu\chi}\left\{\frac{\partial\varphi}{\partial n} - i\nu\varphi[\sin\theta(\cos(n,\xi)\cos\psi + \cos(n,\eta)\sin\psi) + \cos(n,\zeta)\cos\theta]\right\}dS. \tag{1.16}$$

Utilizing formula 1.15, it is possible to find the magnitude of the energy of radiation carried away by the acoustical waves per unit time. This energy may be calculated by examining the effect of pressure forces on a sphere having radius R with the center at the origin of the coordinates. We have

$$N = \int_0^{2\pi} d\psi \int_0^{\pi} p\frac{\partial\Phi}{\partial R} R^2 \sin\theta\, d\theta, \tag{1.17}$$

where p is the pressure exerted by the oscillations of the body,

$$p = -\rho\frac{\partial\Phi}{\partial t}. \tag{1.18}$$

Let us calculate the average amount of energy carried away by the acoustical waves during the period of oscillation $2\pi/k$. To this end we note that if we have two quantities varying in accordance with the harmonic law, i. e., $u = u'e^{ikt}$ and $v = v'e^{ikt}$, the average value of the product of the real parts of these quantities during the period $2\pi/k$ is represented by the formula

$$(uv)_{cp} = \frac{k}{2\pi}\int_0^{2\pi/k} u\,v\,dt = \frac{u\bar{v} + \bar{u}v}{4} = \mathrm{Re}\frac{u\bar{v}}{2} \tag{1.19}$$

where the line over a letter denotes the usual transition to a complex conjugate quantity.

Employing these values and proceeding to the limit where $R \to \infty$, we obtain the following formula for the energy of radiation

$$N_{cp} = \frac{\rho k\nu}{32\pi^2}\int_0^{2\pi} d\psi \int_0^{\pi} |Q(\nu, \theta, \psi)|^2 \sin\theta\, d\theta. \tag{1.20}$$

~~RESTRICTED~~

[redacted]

STAT

The function $Q(\nu,\theta,\psi)$ is linearly related to the complex amplitude of the forward and angular velocities and to the amplitude of the velocity of deformation. Therefore, the energy of the radiation is quadratic in form with reference to these amplitudes.

II. GENERALIZED CONNECTED MASSES AND COEFFICIENTS OF DEFORMATION

It has been established above that the problem of the determination of the disturbed motion of a liquid resolves itself into a determination of the function φ (x,y,z), which satisfies the conditions (1.7 and 1.9). In view of the linearity of the problem, it is possible to obtain

$$\varphi = \sum_{m=1}^{6} \varphi_m u_m + \varphi_0$$

or

$$\varphi = \Phi_1 V + \Phi_2 \omega + \varphi_0, \qquad (2.1)$$

where the vectors $\vec{\Phi}_1$, and Φ_2 have projections on the axes of the coordinates $\varphi_1, \varphi_2, \varphi_3$ and $\varphi_4, \varphi_5, \varphi_6$ respectively. On the surface of the body we have the conditions

$$\frac{\partial \Phi_1}{\partial n} = n, \quad \frac{\partial \Phi_2}{\partial n} = r_0 \times n, \quad \frac{\partial \varphi_0}{\partial n} = n.v_g \text{ on } S \qquad (2.2)$$

In the whole mass of fluid each of the functions φ_m satisfies the wave equation:

$$\frac{\partial^2 \varphi_m}{\partial x^2} + \frac{\partial^2 \varphi_m}{\partial y^2} + \frac{\partial^2 \varphi_m}{\partial z^2} + \nu^2 \varphi_m = 0 \; (m = 0, 1, 2, \ldots, 6) \qquad (2.3)$$

and, finally, these functions satisfy the principle of radiation

$$\lim_{R \to \infty} R \left(\frac{\partial \varphi_m}{\partial R} + i \nu \varphi_m \right) = 0. \qquad (2.4)$$

From the general analysis above it follows that with large values of R, the asymptotic character of the function φ (x,y,z) is determined by the formula:

$$\varphi_m (x, y, z) = -Q_m (\nu, \theta, \psi) \frac{e^{-i\nu R}}{4\pi R} + O\left(\frac{1}{R^2}\right), \qquad (2.5)$$

where

$$Q_m (\nu, \theta, \psi) = \iint_S e^{i\nu\chi} \left\{ \frac{\partial \varphi_m}{\partial n} - i\nu\varphi_m \left[\sin\theta \cos(n,\xi) \cos\psi + \cos(n,\eta) \sin\psi) + \cos(n,\zeta) \cos\theta \right] \right\} dS. \qquad (2.6)$$

[redacted]

RESTRICTED

[redacted]

The functions $\varphi_m(x,y,z)e^{ikt}$ $(m=1,2,\ldots,6)$ are potentials of the velocity of the disturbed motion of the fluid during oscillations of the solid body, the components of the velocities of which have one amplitude, and the function $\varphi_0 e^{ikt}$ is the potential of the velocities of the disturbed movement of the fluid during purely deformatory oscillations of the body.

The functions $\varphi_m(x,y,z)$ are related to the form of the surface S and the parameter ν. The symmetry of the surface S accounts for the corresponding symmetry in the structure of the functions $\varphi_m(x,y,z)$ without relation to the parameter ν. In fact, if the surface S is symmetrical in relation to the plane Oxz, it follows from their border conditions (2.2) that

$$\left(\frac{\partial\varphi_s}{\partial n}\right)_M = \left(\frac{\partial\varphi_s}{\partial n}\right)_{M'} \quad (s=1,3,5); \tag{2.7}$$

$$\left(\frac{\partial\varphi_m}{\partial n}\right)_M = -\left(\frac{\partial\varphi_m}{\partial n}\right)_{M'} \quad (m=2,4,6), \tag{2.8}$$

where M and M' are points on the surface S which are symmetrical in relation to the plane Oxz. From these relations and from formula (1.10) it follows that for those parts of the plane Oxz inside the liquid, the following equations are correct:

$$\frac{\partial\varphi_s}{\partial y} = 0 \quad (s=1,3,5), \quad \varphi_m(x,y,z) = 0 \quad (m=2,4,6). \tag{2.9}$$

At points symmetrical with reference to the plane Oxz, we have

$$\varphi_s(x,y,z) = \varphi_s(x,-y,z) \quad (s=1,3,5), \tag{2.10}$$

$$\varphi_m(x,y,z) = -\varphi_m(x,-y,z) \quad (m=2,4,6). \tag{2.11}$$

If, in addition, the surface S is symmetrical with reference to the plane Oyz, we shall have:

$$\left(\frac{\partial\varphi_s}{\partial n}\right)_P = \left(\frac{\partial\varphi_s}{\partial n}\right)_{P'}, \quad \varphi_s(x,y,z) = \varphi_s(-x,y,z) \quad (s=2,3,4), \tag{2.12}$$

$$\left(\frac{\partial\varphi_m}{\partial n}\right)_P = -\left(\frac{\partial\varphi_s}{\partial n}\right)_{P'}, \quad \varphi_s(x,y,z) = -\varphi_m(-x,y,z) \quad (m=1,5,6) \tag{2.13}$$

where P and P' are points on the surface S which are symmetrical with reference to the plane Oyz.

[redacted]

[redacted]

STAT

Let us investigate the following complex coefficients:

$$G_{jm} = \mu_{jm} - \frac{i}{k}\lambda_{jm} = -\rho \iint_S \varphi_j \frac{\partial \varphi_m}{\partial n}\, dS \; (j, m = 1, 2, \ldots 6) \quad (2.14)$$

It is obvious that these coefficients are related to the geometric properties of the surface S and are functions of the parameter $\varkappa$. A matrix of the 6th order, composed of these coefficients, is symmetrical, i.e.,

$$G_{jm} = G_{mj} \quad (2.15)$$

In fact, applying Green's formula to the region D, outside the surface $S + \Sigma$ where Σ is a sphere of radius R, we obtain

$$\iint_{S+\Sigma} \left(\varphi_j \frac{\partial \varphi_m}{\partial n} - \varphi_m \frac{\partial \varphi_j}{\partial n}\right) dS = -\iiint_D (\varphi_j \Delta \varphi_m - \varphi_m \Delta \varphi_j)\, d\tau.$$

But the right-hand part of this equation, due to equation 2.3, becomes zero. On the other hand, on the basis of the asymptotic formulas 2.4 we have:

$$\lim_{R \to \infty} \iint_\Sigma \left(\varphi_j \frac{\partial \varphi_m}{\partial n} - \varphi_m \frac{\partial \varphi_j}{\partial n}\right) d\Sigma = 0,$$

Hence after the maximum change $R \to \infty$, we shall have

$$\iint_S \left(\varphi_j \frac{\partial \varphi_m}{\partial n} - \varphi_m \frac{\varphi_j}{\partial n}\right) dS = 0, \quad (2.16)$$

and this demonstrates the symmetry of the matrix of the coefficients G_{jm}.

If the plane Oxz is the plane of symmetry of the surface S, out of the 21 constants determining the matrix of the coefficients G_{jm}, only 12 coefficients will differ from zero, i.e., when $j = 1,3,5$ and $m = 2,4,6$, we have $G_{jm} = 0$. If, in addition, the plane Oyz is the plane of symmetry of the surface S, then, except for the diagonal coefficients, only G_{15} and G_{24} will differ from zero. Finally, if the surface S has three mutually perpendicular planes of symmetry, only the diagonal coefficients G_{jj} will differ from zero.

Let us carry on the analysis of the hydrodynamic forces acting on an oscillating body. Using F to denote the principle vector of the hydrodynamic forces acting upon the body and M for the principle moment of these forces in relation to the origin of the coordinates, we shall have the usual formulas:

$$F = -\iint_S p n\, ds, \quad M = -\iint_S p (r_0 \times n)\, dS, \quad (2.17)$$

where

$$p = -\rho \frac{\partial \Phi}{\partial t} = -\rho i k e^{ikt} (\Phi_1 v + \Phi_2 \omega + \varphi_0). \quad (2.18)$$

[redacted]

[redacted]

Putting the expression (2.18) into the formula (2.17) we obtain

$$F = F_0 + F_d, \quad M = M_0 + M_d \tag{2.19}$$

Here F_0 and M_0 are hydrodynamic forces caused by the purely deformatory oscillations of the body, and

$$F_0 = \rho i k e^{ikt} \iint_S \phi_0 n \, dS, \quad M_0 = \rho i k e^{ikt} \iint \phi_0 (r_0 \times n) \, dS; \tag{2.20}$$

F_d and M_d are hydrodynamic forces and the moment caused by the oscillations of the body, investigated as a solid body, and

$$\left.\begin{aligned} F_d &= \rho i k e^{ikt} \iint_S (\phi_1 \cdot v + \phi_2 \cdot \omega) \frac{\partial \phi_1}{\partial n} \, dS, \\ M_d &= \rho i k e^{ikt} \iint (\phi_2 \cdot v + \phi_2 \cdot \omega) \frac{\partial \phi_2}{\partial n} \, dS. \end{aligned}\right\} \tag{2.21}$$

Employing the expression (2.14) for the forces F_d and the moment M_d, we find that:

$$X_m = -\sum_{n=1}^{6} r_{nm} \frac{dU_n}{dt} - \sum_{n=1}^{6} \lambda_{nm} U_n \quad (m = 1, 2, \ldots, 6) \tag{2.22}$$

where X_1, X_2, X_3 are projections of the principal vector of the hydrodynamic forces F_d, and X_4, X_5, X_6 are projections of the principle moment M_d of these forces.

Thus, the hydrodynamic forces produced by the oscillations of the solid body are divided into inertial forces:

$$X'_m = -\sum_{n=1}^{6} \mu_{nm} \frac{dU_n}{dt}, \tag{2.23}$$

and into damping forces:

$$X''_m = -\sum_{n=1}^{6} \lambda_{nm} U_n. \tag{2.24}$$

In determining the law of motion of an oscillating body, the coefficients μ_{nm} of the inertial forces acting on the body play the part of additional masses, of moments of inertia, etc., and the quantities λ_{nm} are coefficients of deformation.

[redacted]

STAT

Therefore, the coefficients μ_{nm} of inertial forces may be called connected masses, this being a generalization of the existing concept of connected masses for an infinite and incompressible fluid. As stated above, the coefficients μ_{nm} and λ_{nm} are related to the geometric properties of the body and are functions of the parameter ν, more precisely, of L/λ, where $\lambda = 2\pi/\nu$ is the length of the radiated acoustical waves and L is the characteristic linear dimension of the body.

In the limited case when $\nu = 0$, i.e., when there are very long radiated waves($\lambda \gg L$), the functions ϕ_m in the vicinity of the body satisfy the Laplace equation:

$$\frac{\partial^2 \phi_m}{\partial x^2} + \frac{\partial^2 \phi_m}{\partial y^2} + \frac{\partial^2 \phi_m}{\partial y^2} = 0, \qquad (2.25)$$

and, consequently, the coefficients G_{jm} are real.

$$G_{jm} = \mu_{jm} = -\rho \iint_S \phi_j \frac{\partial \phi_m}{\partial n} dS, \; \lambda_{jm} = 0. \qquad (2.26)$$

In this case the limited value of the generalized connected masses μ_{jm} coincides with the values of the connected masses during the motion of the body in the incompressible fluid.

In other limited cases when $\nu = \infty$, i.e., when the radiated waves are very short ($\lambda \ll L$), the acoustical radiation proceeds according to the law of geometric acoustics. Each element of the surface S radiates a plane wave in which the velocity of the fluid equals merely the normal component velocity of the given element of the surface S.

Proceeding from considerations of energy, let us express the coefficients of deformation by the energy of radiation carried off by the waves in unit time. Let E be the mechanical energy i.e., the kinetic and potential energy of the volume of the liquid in the region D, included between the surfaces S and Σ. Then, applying the theory of energy of this volume and confining ourselves, for simplification of the radiation, to oscillations of a solid body, we have

$$\frac{dE}{dt} = -F_d \cdot V - M_d \cdot \Omega - N, \qquad (2.27)$$

where N is the energy carried off by the waves over the surface of the sphere Σ during unit time.

The total energy E of the volume of liquid under examination during oscillation is a periodic function of the time; therefore, the average value of the force dE/dt for the period of the oscillations is reduced to zero. Furthermore, it is easy to see that the work of the inertial forces during the period of oscillation is equal to zero. Thus we obtain

$$\frac{1}{2} \sum_{n=1}^{6} \lambda_{nm} |u_n|^2 + \sum_{n/m} \lambda_{nm} Re\,(u_n u_m) = N_{cp}. \qquad (2.28)$$

[redacted]

RESTRICTED

[redacted]

STAT

Here the summation in the second sum extends to all values $n \neq m$ without recurrence. From equation 2.28 we find the formula for λ_{nm} at all values of n and m:

$$\lambda_{nm} = \delta \frac{\partial N_{cp}}{\partial \mathrm{Re}(u_n \bar{u}_m)} (\delta = 1, \text{ if } n \neq m, \text{ and } \delta = 2, \text{ if } n = m), \qquad (2.29)$$

During oscillations of a solid body the function $Q(\gamma, \theta, \psi)$, responsible for the disturbed motion of the fluid, may be expressed linearly by the function $Q_n(\gamma, \theta, \psi)$

$$Q(\gamma, \theta, \psi) = \sum_{n=1}^{6} u_n Q_n(\gamma, \theta, \psi). \qquad (2.30)$$

Consequently, after substituting in (1.20) we obtain

$$\lambda_{nm} = \frac{\rho k \gamma}{16\pi^2} \mathrm{Re} \int_0^{2\pi} d\psi \int_0^{\pi} Q_n(\gamma, \theta, \psi) \bar{Q}(\gamma, \theta, \psi) \sin\theta \, d\theta \qquad (2.31)$$

In particular, when $n = m$, we have:

$$\lambda_{nm} = \frac{\rho k \gamma}{16\pi^2} \int_0^{2\pi} d\psi \int_0^{\pi} |Q_n(\gamma, \theta, \psi)|^2 \sin\theta \, d\theta \qquad (2.32)$$

The whole statement given above refers to the, spatial problem of the acoustical radiation of an oscillating body. In the case of the plane problem of the radiation of the boundary S the problem resolves into the determination of the function $\varphi(x,y)e^{i\omega t}$ fulfilling the conditions:

$$\frac{\partial \varphi}{\partial n} = v(s) \text{ on } s, \qquad (2.33)$$

$$\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} + \gamma^2 \varphi = 0 \text{ outside } s \qquad (2.34)$$

$$\lim_{R \to \infty} \sqrt{R} \left(\frac{\partial \varphi}{\partial R} + i\gamma\varphi \right) = 0 \; (R^2 = x^2 + y^2). \qquad (2.35)$$

The last of these conditions expresses mathematically that, at great distances from the boundary the radiated waves are converted into cylindrical waves.

[redacted]

~~RESTRICTED~~

~~RESTRICTED~~

[redacted]

The general representation of the function φ (x,y) takes the following form [1]:

$$\varphi(x,y)=\frac{i}{4}\int\left(H_0^{(2)}(\gamma r)\frac{\partial\varphi}{\partial n}-\varphi\frac{\partial}{\partial n}H_0^{(2)}(\gamma r)\right)dS, \tag{2.36}$$

where $H_0^{(2)}(\gamma r)$ is Hankel's function of the second type, r is the distance between point (x,y) and point (ξ,η) of the boundary s, i.e.,

$$r=\sqrt{(x-\xi)^2+(y-\eta)^2} \tag{2.37}$$

Employing the asymptotic representation of Hankel's function, we shall, as previously, obtain for the function φ (x,y) the asymptotic formula

$$\varphi(x,y)=\frac{i\,Q^0(\gamma,\theta)}{(8\pi\gamma R)^{1/2}}e^{-i(\gamma R-\frac{\pi}{4})}+O(R^{-\frac{3}{2}}), \tag{2.38}$$

where R and θ are polar coordinates, and

$$Q^0(\gamma,\theta)=$$

$$\int_S e^{i\gamma(\xi\cos\theta+\eta\sin\theta)}\left\{\frac{\partial\varphi}{\partial n}-i\gamma\varphi[\cos\theta\cos(n\xi)+\sin\theta\cos(n,\eta)]\right\}ds, \tag{2.39}$$

Investigating the pressure forces on a circumference having a large radius R, and with the center at the origin of the coordinates, we obtain the following formula for the average magnitude of the energy of the radiation carried away by acoustical waves:

$$N_{cp}=\frac{\rho k}{16\pi}\int_{-\pi}^{+\pi}|Q^0(\gamma,\theta)|^2d\theta. \tag{2.40}$$

It is possible quite analogously to carry on an analysis of the division of the hydrodynamic forces acting on the oscillating boundary s. Considerations of energy lead to the following formula for coefficients of deformation during oscillations of a solid boundary

$$\lambda_{nm}=\frac{\rho k}{8\pi}\int_{-\pi}^{+\pi}Q_n^0(\gamma,\theta)\overline{Q^0_m(\gamma\theta)}\,d\theta, \tag{2.41}$$

$$Q_m^0(\gamma,\theta)=$$

$$\int_S e^{i\gamma(\xi\cos\theta+\eta\sin\theta)}\left\{\frac{\partial\varphi_m}{\partial n}-i\gamma\varphi_m[\cos(n,\xi)\cos\theta+\right.$$
$$\left.+\cos(n,\eta)\sin\theta]\right\}ds, \tag{2.42}$$

[redacted]

~~RESTRICTED~~

STAT

and the functions $Q_m(x,y)$ ($m = 1,2,3$) fulfill the conditions (2.34) and (2.35) and the following conditions on the boundary s:

$$\frac{\partial Q_1}{\partial n} = \cos(n,x), \quad \frac{\partial Q_2}{\partial n} = \cos(n,y),$$

$$\frac{\partial Q_3}{\partial n} = x\cos(n,y) - y\cos(n,x) \qquad (2.43)$$

III. EXAMPLES

Let us examine several examples of calculation for generalized connected masses and coefficients of deformation. Let us first carry out an approximate calculation for coefficients of deformation based on the application of the energy formulas (2.31 and 2.41). Examining the small values of the parameter ν, we may find the first terms of the dissociation of the function $Q_m(\nu,\theta,\psi)$ according to the degrees of ν.

Let us take a symmetrical body with reference to the coordinate planes, oscillating progressively with three degrees of freedom; then, from formulae (2.6), correct up to the terms containing ν^2, we obtain:

$$Q_1 = i\nu \sin\theta\cos\psi\left(V + \frac{\mu_{11}(0)}{\rho}\right),$$
$$Q_2 = i\nu \sin\theta\sin\psi\left(V + \frac{\mu_{22}(0)}{\rho}\right),$$
$$Q_3 = i\nu \cos\theta\left(V + \frac{\mu_{33}(0)}{\rho}\right),$$

where V is the volume of the body, and $\mu_{11}(0)$, $\mu_{22}(0)$ and $\mu_{33}(0)$ are values of the generalized connected masses when $\nu = 0$.

According to formula 2.32 we find:

$$\tilde{\lambda}_{nn} = \frac{\rho k \nu^3}{12\pi}\left[V + \frac{\mu_{nn}(0)}{\rho}\right]^2 \quad (n = 1,2,3). \qquad (3.2)$$

Here the curved mark ($\sim$) over the letter denotes a value of given magnitude at small values of the parameter ν.

Analogous formulas hold true in the rotary oscillations of a body. In the particular case of the arbitrary configuration of a disk during its rotary oscillation around the axes x and y, we have:

$$Q_n(\nu,\theta,\psi) =$$

$$-i\nu\cos\theta\iint_S e^{i\nu\sin\theta(x\cos\psi + y\sin\psi)}(\varphi_n^+ - \varphi_n^-)\,ds \quad (n=4,5), \qquad (3.3)$$

[redacted]

where Q_n^+ is the value of the functions Q_n on the upper side of the disk, and Q_n^- is the value of these functions on the lower side of the disk. Retaining in formula 3.3 only the terms containing $\sqrt{\nu}$, we find:

$$Q_4 = -\nu^2 \sin\Theta\cos\Theta\sin\psi\frac{\mu_{44}(0)}{\rho}, \quad Q_5 = \nu^2\sin\Theta\cos\Theta\cos\psi\frac{\mu_{55}(0)}{\rho},$$

$$\lambda_{nn} = \frac{k\nu^4}{60\pi\rho}\mu_{nn}^2(0) \quad (n=4,5) \qquad (3.4)$$

As an application of formula 2.41 let us examine a plane lamina of infinite span effecting vertical and rotary oscillations. Proceeding as in the previous examples, for the functions Q_2^0 and Q_3^0 we obtain:

$$Q_2^0 = i\nu\cos\Theta\frac{\mu_{22}(0)}{\rho} \qquad Q_3^0 = -\nu^2\cos^2\Theta\frac{\mu_{33}(0)}{\rho}$$

Employing the well-known expression for connected masses of a plane:

$$\mu_{22}(0) = \pi\rho a^2, \quad \mu_{33}(0) = \frac{\pi}{8}\rho a^4,$$

we find that:

$$\lambda_{22} = \frac{1}{8}\rho\pi^2\nu^3 a^4, \quad \tilde{\lambda}_{33} = \frac{3}{2''}\rho\pi^2\nu^4 a^8 \qquad (3.5)$$

Let us now examine some examples of exact calculation for generalized connected masses and coefficients of deformation during acoustical radiation of oscillating bodies in a compressible liquid.

As a first example, let us investigate the acoustical radiation produced by a globe of radius R during its vertical oscillation. The function $\varphi_3(x,y,z)$ corresponding to the oscillations of the globe takes the form:

$$\varphi_3 = -\frac{R^3 e^{i\nu R}}{2-\nu^2R^2+2i\nu R}\left(\frac{1}{r}+i\nu\right)\frac{e^{-i\nu r}}{r}\cos\Theta, \qquad (3.6)$$

where r is the distance between the point P (x,y,z) and the center of the globe and Θ is the angle between the direction of radius r and the axis Oz.

According to formula 2.14 we have:

$$\mu - \frac{i}{k}\lambda = -\rho\iint_S \varphi_3\cos(n,z)\,dS,$$

[redacted]

[redacted]

STAT

[redacted]

and, consequently, we find the expressions $\mu(\nu)$ and $\lambda(\nu)$, represented in Figure 1:

$$\mu(\nu)=\mu(0)\frac{4+2\nu^2R^2}{4+\nu^2R^2},$$

$$\lambda=\frac{4\tilde{\lambda}}{4+\nu^2R^2}\left(\mu(0)=\frac{2}{3}\pi\rho R^3;\ \tilde{\lambda}=\frac{k}{3}\pi\rho R^3\nu^3R^3\right) \qquad (3.7)$$

In a like manner it is possible to find the exact value of the connected mass and of the coefficient of deformation during oscillations of a cylinder in the direction of axis Oy. For the function $\varphi(x,y)$, responsible for the oscillations of the cylinder, we find that:

$$\varphi_2=\frac{1}{\nu\dfrac{\partial^2 H_0^{(2)}}{\partial x^2}}\ \frac{\partial H_0^{(2)}(\nu r)}{\partial(\nu r)}\sin\Theta_{(x=\nu R)}, \qquad (3.8)$$

and, according to formula 2.14, we obtain the expressions for $\mu(\nu)$ and $\lambda(\nu)$, represented in Figure 2:

$$\mu(\nu)=\mu(0)\frac{J_1(J_1-xJ_0)+N_1(N_1-xN_0)}{(J_1-xJ_0)^2+(N_1-xN_0)^2},$$

$$\lambda=\tilde{\lambda}\frac{2}{\pi x^2}\frac{N_1(J_1-xJ_0)-J_1(N_1-xN_0)}{(J_1-xJ_0)^2+(N_1-xN_0)^2} \qquad (3.9)$$

$$\left(\mu(0)=\rho\pi R^2,\ \tilde{\lambda}=\frac{k}{2}\rho\pi^2R^2x^2,\ x=\nu R\right)$$

Here $J_p(x)$ is Wessel's function and $N_p(x)$ is Neumann's function.

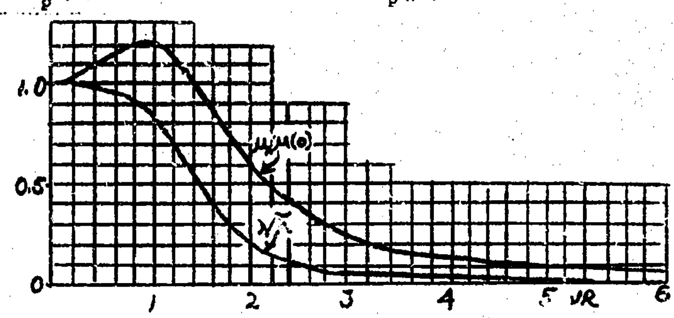

Figure 1. The Connected Mass and the Coefficient of Deformation During Oscillations of a Globe

If the wave functions of Lame and Mathieu and the function of a parabolic cylinder are employed, it is possible to investigate several more general problems, for example, the problem of acoustical radiation arising during the oscillation of an ellipsoid, an elliptic or parabolic cylinder.

[redacted]

STAT

[redacted]

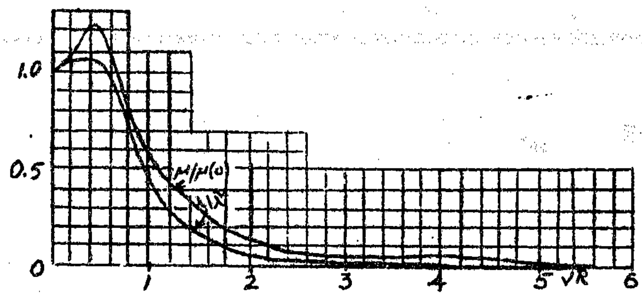

Figure 2. The Connected Mass and the Coefficient of Deformation of a Cylinder

As an application of the above-mentioned functions let us examine the simplest example of acoustical radiation produced by a plane lamina of infinite span. In this case the function $Q(x,y)$, responsible for the vertical and rotary oscillations of the lamina, is determined according to the formula:

$$Q(x,y) = V Q_1(x,y) + \Omega Q_2(x,y), \tag{3.10}$$

where V and Ω are the complex amplitudes of the corresponding vertical and angular velocity and the functions Q_1 and Q_2 fulfill the wave equation and the border conditions on the plane.

$$\frac{\partial Q_1}{\partial y} = 1; \frac{\partial Q_2}{\partial y} = x \quad \text{when } y = 0, |x| \le a \tag{3.11}$$

Introducing the elliptical coordinates $x + iy = a\,ch(\xi + i\eta)$, we can represent the wave equation in these coordinates in the form:

$$\frac{\partial^2 Q}{\partial \xi^2} + \frac{\partial^2 Q}{\partial \eta^2} + \nu^2 a^2 (ch^2 \xi - \cos^2 \eta) Q = 0 \tag{3.12}$$

The border conditions (3.11) in the coordinates ξ and η will take the form:

$$\frac{\partial Q_1}{\partial \xi} = a \sin \eta, \frac{\partial Q_2}{\partial \xi} = \frac{1}{2} a^2 \sin 2\eta \quad \text{when } \xi = 0 \tag{3.13}$$

From considerations of symmetry it also follows that

$$Q_1 = Q_2 = 0 \quad \text{when } y = 0, |x| > a \ (\eta = 0, \pm \pi). \tag{3.14}$$

Let us represent the functions Q_1 and Q_2 in terms of dissociation in series according to Mathieu and Hankel's functions (2):

[redacted]

RESTRICTED

[redacted]

$$\left.\begin{aligned}\varphi_1 &= \sum_{n=1}^{\infty} a_{2n+1}\, Se_{2n+1}(\xi)\, se_{2n+1}(\eta),\\ \varphi_2 &= \sum_{n=1}^{\infty} a_{2n}\, Se_{2n}(\xi)\, se_{2n}(\eta).\end{aligned}\right\} \qquad (3.15)$$

Here $se_{2n+1}(\eta)$ and $se_{2n}(\eta)$ are the periodic odd functions of Mathieu with the following normalization

$$\int_{-\pi}^{+\pi} se_m^2(\eta)\, d\eta = \pi,$$

$Se_{2n+1}(\xi)$ and $Se_{2n}(\xi)$ are the corresponding Hankel Mathieu's functions which fulfill condition (2.15), and constants a_{2n+1} and a_{2n} are the coefficients of dissociation, which are to be determined.

It is easy to see that the dissociations (3.15) fulfill condition (3.14). By employing the orthogonal Mathieu function and conditions (3.13), we obtain the following expressions for the coefficients a_{2n+1} and a_{2n}:

$$a_{2n+1} = \frac{1}{\pi Se'_{2n+1}(0)} \int_{-\pi}^{+\pi} \sin n\eta\, se_{2n+1}(\eta)\, d\eta = \frac{1}{\pi Se'_{2n+1}(0)} B_1(2n+1), \qquad (3.16)$$

$$a_{2n} = \frac{1}{2\pi Se'_{2n}(0)} \int_{-\pi}^{+\pi} \sin 2\eta\, se_{2n}(\eta)\, d\eta =$$

$$= \frac{1}{2 Se'_{2n}(0)} B_2(2n); \quad Se'_m(0) = \left(\frac{d\, Se_m(\xi)}{d\xi}\right)_{\xi=0},$$

where B_1 and B_2 are the first coefficients of dissociation in the Fourier series corresponding to Mathieu's function $se_{2n+1}(\xi)$ and $se_{2n}(\eta)$ (Ayns's designation) [2].

For the connected masses and coefficients of deformation we have the formulas:

$$\mu_1 - \frac{i}{K} = -\rho a \int_{-\pi}^{+\pi} \varphi_1 \sin\eta\, d\eta, \quad \mu_2 - \frac{i}{K}\lambda_2 = -\frac{1}{2}\rho a^2 \int_{-\pi}^{+\pi} \varphi_2 \sin 2\eta\, d\eta \qquad (3.17)$$

on the basis of which we find that:

$$\mu_1 - \frac{i}{K}\lambda_1 = -\pi\rho a^2 \sum_{n=1}^{\infty} \frac{Se_{2n+1}(0)}{Se'_{2n+1}(0)} B_1^2(2n+1), \qquad (3.18)$$

[redacted]

~~RESTRICTED~~

~~RESTRICTED~~

[redacted]

STAT

$$\mu_2 - \frac{i}{k}\lambda_2 = -\frac{\pi}{4}\sum_{n=1}^{\infty}\frac{Se_{2n}(0)}{Se'_{2n}(0)}B_2^2(2n). \qquad (3.19)$$

The coefficients B_1 and B_2 decrease very rapidly with an increase in the number n; consequently, to calculate $\mu_{1,2}$ and $\lambda_{1,2}$ it is possible to take a small number of terms in the dissociations (3.18) and (3.19).

BIBLIOGRAPHY

1. H. Lamb, Hydrodynamics, 1932.

2. M. D. O Strutt, Lame and Mathieu Functions and Functions Related to them in Physics and Technology, ONTI, 1935.

- E N D -

[redacted]

~~RESTRICTED~~

Sanitized Copy Approved for Release 2011/06/29 : CIA-RDP80-00809A000600200333-1